Comparison of calculated and measured performance of diffused channel-waveguide couplers

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Diffused channel-waveguide couplers were fabricated on z-cut y-propagating LiNbO$_3$ crystals by the indiffusion of titanium. Both mode spot sizes and coupling lengths were measured. The fields and coupling lengths were computed for both TM and TE polarizations. Good agreement was obtained between the measured and computed mode spot sizes and coupling lengths. It is concluded that the calculation method could aid in the design of such couplers.

1. INTRODUCTION

Optical waveguides produced by the diffusion of titanium into lithium niobate crystals are the basis for a variety of integrated optics devices with potential applications to optical communications. Among these are directional couplers, which have been used to fabricate low-loss optical switches, high-speed modulators, polarization converters, and wavelength filters. The development of these devices is normally carried out by trial and error with little reliance on theory. Since this method of development can be both tedious and expensive, a clear need exists for an accurate computational method to guide design.

To be useful, such a method should be capable of predicting coupler performance and field characteristics from a knowledge of the coupler geometry and fabrication conditions alone. The necessary steps in the accomplishment of this task are first, the computation of the two-dimensional (2-D) index profiles for the waveguides from initial titanium strip geometry, diffusion data, and refractive-index variation with titanium concentration and second, the accurate computation of propagation constants and modal fields for the combined set of profiles. Since there are uncertainties in the required data, it is essential that the second step be performed with as few mathematical simplifications and approximations as possible. This of necessity almost dictates using a numerical method.

In this paper we present some experimental and theoretical results obtained in an effort to model laboratory couplers from the kinds of data mentioned above. Computations were based on fabrication conditions. The diffusion data were measured in house to be directly applicable to the waveguides used in the experiment, and the data for $dn_n/dc$ and $dn_e/dc$, the rates of change of the ordinary and extraordinary refractive indices with titanium concentration, were taken from Ref. 7. With slight adjustments of the assumed values of the undiffused titanium thickness and $dn_n/dc$, it was possible to obtain an accurate description of beam shapes and sizes and coupling length as a function of waveguide separation for both TE and TM polarizations. Because of uncertainties in the measured data, the need for these slight adjustments is not unexpected. The accuracy of the calculated results gives confidence that the computational method can aid in the design of diffused channel couplers, when applicable data is available.

Channel waveguides and couplers have been treated with a variety of theoretical methods. Marcottii applied perturbation theory to a coupler formed from rectangular waveguides with uniform index cores. Hocker and Burns used the effective index method on a single waveguide with a 2-D diffusion profile. The variational method was applied to a 2-D diffusion profile by Taylor and Korotky. The WKB method was applied to a 1-D coupler profile constructed from index measurements at the surface of the crystal. Jain et al. applied the variational method to a channel waveguide coupler with an approximate 2-D diffusion profile. No reference to polarization was made, however, and agreement with experiment was achieved by adjusting both the height and the width of the profile. To our knowledge, the present work represents the first attempt to compute the performance of diffused-channel couplers from fabrication conditions alone.

Yeh et al. modeled guided-wave propagation in two identical coupled fibers with the help of numerical solutions to the paraxial-wave equation. These solutions were used to display qualitative features of the periodic transfer of energy between the waveguides as a function of propagation distance. It is also possible, however, to compute the propagation constants and eigenfunctions for a coupler or a general class of 2-D profiles by performing Fourier transforms on numerical solutions to the paraxial-wave equation. Applied to a coupler, this method permits accurate computation of the coupling length from the relation $l = \pi/\Delta\beta$, where $\Delta\beta$ is the difference between the propagation constants of the even- and odd-symmetry modes. It also permits computation of the mode eigenfunctions and the fraction of energy transferable from one waveguide to the other. The accuracy of this method, which has been called the propagating-beam method, has been established by comparing results with analytic solutions.
We have employed the propagating-beam method to analyze the couplers described in this paper.

The paper is organized as follows. The experiment is described in Section 2. In Section 3, the relevant electromagnetic theory is outlined. In particular, Helmholtz and paraxial-wave equations that take account of medium birefringence are presented, and the spectral techniques used in connection with the propagating-beam method to compute propagation constants and mode eigenfunctions are reviewed. The computation of the diffused index profiles is described and the necessary data are presented in Section 4. Experimental and theoretical results for beam properties and coupling lengths are compared in Section 5. Section 6 is a summary, and Appendix A deals with the effect of incomplete energy transfer between couplers on the measurement of coupling length.

2. EXPERIMENT

The waveguide couplers used in the experiments were fabricated on z-cut y-propagating LiNbO$_3$ crystals. The waveguide patterns were produced by standard photolithographic techniques. Titanium was deposited by thermal evaporation, and its thickness was measured with a profilometer. The deposited strips were 6.3 $\mu$m in width and 0.095 $\mu$m in thickness. The deposited metal was allowed to indiffuse for 6 h at a temperature of 1050°C under flowing O$_2$ and H$_2$O to inhibit Li$_2$O outdiffusion. A series of directional couplers was fabricated with the gap between waveguides varying from 3 to 7 $\mu$m and coupling lengths varying from 200 $\mu$m to 5 mm. The waveguide configuration of the couplers is indicated in Fig. 1.

A schematic diagram of the apparatus for measuring coupling lengths is shown in Fig. 2. Polarized light was launched into the coupler from a single-mode fiber. A Babinet-Soleil compensator was used to correct for fiber stress birefringence. A 40X microscope objective was used to separate and magnify the two mode patterns of the output waveguides. The relative intensity of the output light was measured with a germanium detector, a current preamplifier, and a lock-in amplifier. Mode dimensions were measured with an IR vidicon calibrated to correct for nonlinear response. The position of the peak power of the mode with respect to the crystal surface was measured by illuminating the exit face of the crystal with radiation redirected from the laser. A beam splitter recovered both the mode image and the image of the LiNbO$_3$ crystal surface.

The characteristic coupling length $l$ (see Fig. 1) of a waveguide pair was extracted by using the expression for the interguide power-transfer efficiency:

$$\eta = \frac{P_A}{P_A + P_B} = \sin^2 \left( \frac{\pi L}{2l} + \phi \right),$$

where $L$ is the length of the coupler and the phase $\phi$ results from energy coupled between waveguides in the tapered section leading to the coupler.

Equation (1) can be rewritten in the form

$$\frac{2}{\pi} \sin^{-1} \eta^{1/2} = \frac{L}{l} + \frac{2\phi}{\pi},$$

and the slope of a plot of $(2/\pi) \sin^{-1} \eta^{1/2}$ versus $L$ is used to determine the value of $1/l$. A sample plot for a set of couplers with interwaveguide gap $d = 3 \mu$m is shown in Fig. 3.

A consequence of Eq. (1), which is based on coupled-mode theory, is that the energy transfer between waveguides over a coupling length is complete. This condition can be ap-
approximately fulfilled only when the waveguides are far apart. The incompleteness of the energy transfer is expected because the evanescent field emanating from one waveguide must penetrate the other waveguide if coupling is to take place. Thus neither waveguide can contain all the energy, and energy transfer can never be complete. In Appendix A, the consequences of incomplete energy transfer are analyzed, and in particular the use of Eq. (2) is justified when energy transfer is less than complete.

3. THEORY

A. Wave Equations and Method of Solution

In the limit of weak guidance, which applies to the waveguiding structures considered in this paper, the electric field can be assumed to be polarized either parallel (TE) or perpendicular (TM) to the crystal surface. The TE or ordinary wave mode is described by the Helmholtz equation

$$\nabla^2 E_x + \frac{\partial^2 E_x}{\partial z^2} + k^2 n_0^2(x, z) E_x = 0, \quad (3)$$

where $k$ is the free-space wave number and $n_0(x, z)$ is the ordinary refractive index. The TM or extraordinary wave mode is described by the Helmholtz equation

$$\nabla^2 E_z + \frac{\partial^2 E_z}{\partial z^2} + k^2 n_e^2(x, z) E_z = 0, \quad (4)$$

where $n_e(x, z)$ is the extraordinary refractive index. The refractive indices $n_0(x, z)$ and $n_e(x, z)$ can be expressed as

$$n_0(x, z) = \bar{n}_0 + \Delta n_0(x, z),$$

$$n_e(x, z) = \bar{n}_e + \Delta n_e(x, z), \quad (5)$$

where $\bar{n}_0$ and $\bar{n}_e$ are the refractive indices of the LiNbO$_3$ substrate and $\Delta n_0(x, z)$ and $\Delta n_e(x, z)$ are the changes in the index that are due to the diffused titanium.

Let $E_x(x, y, z)$ and $E_z(x, y, z)$ be expressed as

$$E_x(x, y, z) = \xi_x(x, y, z) \exp(-i\bar{n}_0 y),$$

$$E_z(x, y, z) = \xi_z(x, y, z) \exp(-i\bar{n}_e y). \quad (6a)$$

The paraxial-wave equations for $\xi_x(x, z)$ and $\xi_z(x, z)$ can be derived by substituting Eqs. (6a) and (6b) into Eqs. (3) and (4) and neglecting $\frac{\partial^2\xi_x}{\partial y^2}$ and $\frac{\partial^2\xi_z}{\partial y^2}$. The resulting equations are

$$2in_0k \frac{\partial \xi_x}{\partial y} = \frac{\partial^2 \xi_x}{\partial x^2} + \frac{\partial^2 \xi_x}{\partial z^2} + k^2(\Delta n_0^2(x, z) - \bar{n}_0^2) \xi_x, \quad (7a)$$

$$2in_0k \frac{\partial \xi_z}{\partial y} = \frac{\partial^2 \xi_z}{\partial x^2} + \frac{\partial^2 \xi_z}{\partial z^2} + k^2(\Delta n_e^2(x, z) - \bar{n}_e^2) \xi_z. \quad (7b)$$

The eigenfunctions in Eq. (9) are identical to those of the corresponding Helmholtz equations, and the Helmholtz propagation constants $\beta_{sn}$ are related to the paraxial propagation constants $\beta_{sn}'$ by the relation

$$\beta_{sn} = \frac{\bar{n}_e k (1 + 2\beta_{sn}'/\bar{n}_e k)^{1/2}}, \quad (10)$$

where the index $s$ is used to represent $x, z,$ and $o, e$ interchangeably. Conversely,

$$\beta_{sn}' = \frac{\beta_{sn}^2 - \bar{n}_e^2 k^2}{2\bar{n}_e k}. \quad (11)$$

Thus knowledge of the paraxial-equation eigenvalues and eigenfunctions immediately gives the eigenfunctions and eigenvalues of the corresponding Helmholtz equation and vice versa.

To apply the propagating-beam method, it is necessary to solve Eqs. (7a) and (7b) numerically by the split-operator Fast-Fourier-transform method and to compute numerically the correlation function

$$\mathcal{P}_1(y) = \int \xi_1^*(x, 0, z) \xi_2(x, y, z) \, dx \, dz. \quad (12)$$

where $V$ is the propagation distance in the numerical solution, and the numerical Fourier transform of the product is computed, the result is

$$\mathcal{P}_1(\beta) = \sum_n W_n \mathcal{L}_1(\beta - \beta_{sn}'), \quad (14)$$

where

$$W_n = |A_{sn}|^2 \quad (15)$$

and

$$\mathcal{L}_1(\beta - \beta_{sn}') = \frac{\exp[i(\beta - \beta_{sn}') Y] - 1}{i(\beta - \beta_{sn}') Y} - \frac{1}{2} \left[ \frac{\exp[i(\beta - \beta_{sn}') Y + 2\pi] - 1}{i[(\beta - \beta_{sn}') Y + 2\pi]} + \frac{\exp[i(\beta - \beta_{sn}') Y - 2\pi] - 1}{i[(\beta - \beta_{sn}') Y - 2\pi]} \right]. \quad (16)$$

The eigenvalues $\beta_{sn}'$ are determined by locating the resonant peaks of the spectrum $\mathcal{P}_1(\beta)$, and the mode eigenfunctions are evaluated by computing numerically the integrals

$$u_{sn}(x, y) = \text{const} \times \int_0^Y \xi_s(x, y, z) w(y) \exp(i\beta_{sn}' y) \, dy. \quad (17)$$

B. Boundary Conditions

The large refractive-index discontinuity at the air–crystal interface poses a problem for the propagating-beam method, which is valid only for weak refractive-index gradients. One can assume to an excellent approximation, however, that the air–crystal interface is a perfectly reflecting boundary. Two simple operations serve to impose the reflecting-
boundary condition without complicating the application of the method. Let the air-crystal interface be the plane \( z = 0 \).

First, we reflect the refractive-index distribution about \( z = 0 \), i.e., we let \( n_s(x, -z) = n_s(x, z) \), where \( s = z, 0 \), and air occupies the region of negative \( z \); and second, we launch fields at \( y = 0 \) that have the form

\[
\varepsilon_s(x, 0, -z) = -\varepsilon_s(x, 0, z),
\]

which guarantees that the field will remain an odd function of \( z \) for all values of the propagation distance \( y \).

We shall also be interested in exciting the two modes of the coupler that are either even or odd with respect to a reflection about a plane halfway between the two waveguides. Let that plane be \( x = 0 \). The launched field must consequently also obey either of the conditions

\[
\varepsilon_s(x, 0, z) = f(x - x_0, z) \pm f(x + x_0, z), \quad z \geq 0,
\]

where \( x_0 \) and \(-x_0\) are the coordinates of the centers of the two wave guides and \( f(x, z) \) is an odd function of \( z \).

4. REFRACTIVE-INDEX PROFILE

For diffused channel waveguides, the incremental refractive indices \( \Delta n_t \) and \( \Delta n_e \) can be expressed in terms of the diffused titanium concentration \( c(x, z) \) expressed as a fraction of the atom density present in titanium metal.

The diffused titanium-ion distribution for a single waveguide can be written as

\[
\Delta c(x, z) = \frac{1}{2} c_0 \exp \left( -\frac{z^2}{4D_t t} \right) \text{erf} \left( \frac{w/2 + x}{2(D_t t)^{1/2}} \right) + \text{erf} \left( \frac{w/2 - x}{2(D_t t)^{1/2}} \right),
\]

where

\[
c_0 = \tau / (\pi D_t t)^{1/2}.
\]

In Eqs. (20) and (21), \( D_t \) is the bulk-diffusion coefficient, \( D_e \) is the surface-diffusion coefficient, \( t \) is the diffusion time, and \( \tau \) and \( w \) are the thickness and the width, respectively, of the evaporated titanium strip before diffusion. The normalization coefficient \( c_0 \) is chosen so that

\[
\int_0^\infty dz \int_{-\infty}^\infty \Delta c(x, z) dx = \tau w.
\]

Equation (20) thus gives the relative concentration distribution, provided that the density of the deposited titanium layer is the same as that of titanium metal. It is possible, however, that the density of the deposited layer could be less than that of titanium metal. In that event it would be necessary to reduce the measured value of \( \tau \) to reflect this reduced density before applying Eqs. (20) and (21).

The bulk diffusion coefficient \( D_t \) was measured using a planar layer of titanium 300 nm thick evaporated onto a congruent \( z \)-cut crystal of LiNbO\(_3\) (48.6 mol % Li\(_2\)O). The metal thickness was determined with a profilometer. Indiffusion took place at 1050°C under flowing \( \text{O}_2 \) and \( \text{H}_2\text{O} \) vapor, and microprobe analysis was performed on the polished ends of the crystal to determine the diffused titanium concentration profile. The latter was fitted to a half-Gaussian curve, and the metal diffusivity was computed from the half-width. The value of \( D_t \) obtained was \((1.0 \pm 0.1) \times 10^{-4} \text{ mm}^2/\text{sec} \). The corresponding diffusion length is \((4D_t t)^{1/2} = 2.94 \text{ mm} \). Fukuoka and Noda\(^{23}\) observed an anisotropy in the diffusion coefficients for LiNbO\(_3\). For our experimental conditions, their data imply that \( D_t = 2.19 \text{ mm}^2/\text{sec} \) and \( D_e = 1.7 \text{ mm}^2/\text{sec} \), or \((4D_t t)^{1/2} = 4.35 \text{ mm} \) and \((4D_e t)^{1/2} = 3.83 \text{ mm} \). Because of the closeness of these diffusion lengths and possible differences in stoichiometry between their crystals and ours, we have ignored diffusion anisotropy.

From the relative concentration distribution of Eq. (20), the incremental refractive indices can be computed using

\[
\Delta n_t(x, z) = \frac{dn_o}{dc} \Delta c(x, z),
\]

\[
\Delta n_e(x, z) = \frac{dn_e}{dc} \Delta c(x, z).
\]

The following expressions provide an accurate representation of the measurements reported in Ref. 7:

\[
\frac{dn_o}{dc} = \begin{cases} 
0.89 & c \leq 0.003 \\
0.47 & 0.003 \leq c \leq 0.007 \\
0.32 & c > 0.007
\end{cases}
\]

and

\[
\frac{dn_e}{dc} = 0.625.
\]

Because of the differences in the preparation of the material samples used in the measurements described in Ref. 7 and in our experiments, there is an uncertainty in the applicability of Eqs. (24) and (25) to our couplers. Thus, not surprisingly, we were able to obtain consistent agreement between theory and experiment by increasing the value of \( dn_o/dc \) by 10% and by decreasing all values of \( dn_e/dc \) by 10%. An equivalent adjustment as far as computation of the index profiles is concerned is reduction of the measured \( \tau \) by 10% to account for less-than-normal density in the deposited titanium layer and an increase in the value of \( dn_e/dc \) by 20% to

\[
\frac{dn_e}{dc} = 0.764.
\]

In the remainder of the paper the adjustment of parameters is given the latter interpretation.

The values for the substrate refractive indices employed in the computations were \( n_o = 2.1567 \) and \( n_e = 2.237 \) at a wavelength of 1.32 \( \mu \text{m} \).

5. COMPARISON OF EXPERIMENT AND THEORY

A. Mode-Spot Size

Figure 4 shows the original titanium strip, constant refractive-index contours, and isointensity contours for the TM-mode field. The isointensity contours are plotted beside a isointensity contour map for the calculated field. The close resemblance between the calculated and the measured beam shape.

The calculated and the measured mode dimensions are compared for the unadjusted and adjusted values of \( \tau \) and \( dn_e/dc \) in Tables 1 and 2. The best agreement in Table 1 is
FIG. 4. Single diffused channel waveguide: (a) undiffused titanium strip, (b) contours of constant refractive index, and (c) isointensity contours of mode field.

Fig. 5. Experimental and theoretical mode spot for single diffused channel waveguide.

Table 1. Mode-Spot Sizes Measured and Calculated from Unadjusted Parameters

<table>
<thead>
<tr>
<th>Amplitude Width</th>
<th>TE Mode</th>
<th>TM Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Theory</td>
</tr>
<tr>
<td>Parallel</td>
<td>9.12</td>
<td>8.84</td>
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<tr>
<td>Normal</td>
<td>5.97</td>
<td>5.75</td>
</tr>
<tr>
<td>Depth</td>
<td>2.25</td>
<td>2.50</td>
</tr>
</tbody>
</table>

* Full-width 1/e amplitude width parallel and normal to surface and depth in micrometers.

Table 2. Mode-Spot Sizes Measured and Calculated by Adjusting Measured \( \tau \) Downward by 10% and \( dn_e/dc \) of Ref. 7 Upward by 20%

<table>
<thead>
<tr>
<th>Amplitude Width</th>
<th>TE Mode</th>
<th>TM Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Theory</td>
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<td>6.05</td>
</tr>
<tr>
<td>Depth</td>
<td>2.25</td>
<td>2.50</td>
</tr>
</tbody>
</table>

* Full-width 1/e amplitude width parallel and normal to surface and depth in micrometers.

for the TE-mode sizes, which agree to within 3 to 4%. The sizes for the TM mode, on the other hand, agree to only 11.5 and 18.7%, respectively. The discrepancy in the depth measurements may in part be attributable to the difficulty of determining experimentally the position of the crystal edge.

It is seen from Table 2 that a reduction of the measured value of \( \tau \) by 10% brings the size agreement for the TE mode to within 1% or less. An increase by about 20% in \( dn_e/dc \) improves the agreement for the TM sizes to 6 and 8%, respectively. At the same time, this value of \( dn_e/dc \) gives good agreement between measured and computed coupling lengths for the same polarization.

B. Coupler Performance

Figure 6 shows the undiffused strip geometry, the diffused refractive-index contours, and the isointensity contours of the even- and odd-symmetry modes corresponding to the waveguide separation \( d = 2.75 \mu m \). Figure 7 shows the isointensity contours for the even- and odd-symmetry modes and their sum and difference. The contours for the mode combinations indicate the amount of energy that can be transferred from one waveguide to the other. For both mode combinations, the bulk of the energy lies on one side of the origin. The small portion of energy that crosses the origin represents energy that cannot be transferred.

Figures (8a) and (8b) show the mode spectra for an individual waveguide and for the corresponding coupler, respectively. These spectra were used to determine the propagation constants. Only those peaks to the left of the origin correspond to bound modes. The remaining modes are leaky modes and need not be considered. The separation of the peaks in Fig. (8b) clearly indicates the value of \( \Delta \beta' \), which determines the coupling length through the relation \( l = \pi/\Delta \beta' \).

Figure 9 shows the evolution of the field intensity as a function of propagation distance when the mode of a single waveguide is injected into one arm of the coupler and propagated one coupling length. At this distance, 86% of the energy
Fig. 7. Isointensity contours for modes of diffused channel-waveguide couplers: (a) even-symmetry mode, (b) odd-symmetry mode, (c) sum of even- and odd-symmetry modes, and (d) difference of even- and odd-symmetry mode. has crossed from one waveguide to the other. This is close to the 88% that would be transferred if the field initially contained equal weights of the two normal modes (see Fig. 7).

The sensitivity of computed coupling lengths to variations in diffusion length is demonstrated in Fig. 10. Plotted is the coupling length $l$ versus interguide separation $d$, computed for the profile function [Eq. (20)] and three different sets of diffusion data. Two plots were computed for a single diffusion constant $D_x = D_z$. The remaining plot was computed with $D_x$ and $D_z$ computed from data in Ref. 23. It is seen from Fig. 10 that variation of the diffusion length produces a parallel shift of the $l$ versus $d$ plots.

Variation of $\tau$ and/or $dn_e/dc$, on the other hand, produces a rotation in the $l$ versus $d$ plots, which is evident in Fig. 11. These plots were computed for the TM polarization with the unadjusted and adjusted values of $\tau$ and $dn_e/dc$. These parameters have a simple multiplicative effect on the profile function [Eq. (20)]; diffusion lengths have a more complicated influence.

![Fig. 8](image_url)  
**Fig. 8.** Peaks in mode spectra identify propagation constants: (a) single waveguide and (b) coupler.

![Fig. 9](image_url)  
**Fig. 9.** Power transfer between waveguides in a diffused channel-waveguide coupler. Mode of single waveguide is launched into one arm. At the end of one coupling length $l$, 86% of power is transferred.

![Fig. 10](image_url)  
**Fig. 10.** Sensitivity of computed coupling length to diffusion length. Lower curve was calculated with the anisotropic diffusion lengths of Ref. 23.

![Fig. 11](image_url)  
**Fig. 11.** Sensitivity of computed coupling length for TM polarization to $\tau$ and $dn_e/dc$. Lower curve is for unadjusted values. Upper curve is for 10% downward adjustment of $\tau$ and upward adjustment of 20% in $dn_e/dc$. 
Figure 12 shows measured and computed coupling lengths as a function of waveguide separation for both TE and TM polarizations. Computations were made with the adjusted values of $\tau$ and $dn_e/dc$. Each pair of experimental coupling lengths required the fabrication of 14 couplers. Agreement between calculated and measured coupling lengths is good overall. Of particular interest is the waveguide separation that gives the same TM and TE coupling lengths, since the behavior of the corresponding coupler is independent of polarization. Here the agreement is even better.

6. SUMMARY AND CONCLUSION

We have modeled the performance of diffused channel-waveguide couplers from fabrication conditions alone, applying data measured for the specific laboratory couplers wherever possible and published data otherwise. With only small adjustments to two parameters, either $dn_o/dc$ and $dn_e/dc$ or $\tau$ and $dn_e/dc$, we obtained good agreement with experimental results. Specifically, we found good agreement between computations and measurements of mode-spot size and shape and coupling length as a function of waveguide separation for both TE and TM polarizations. These results are evidence of the overall accuracy of the computational method and lead to confidence that it can assist in the design of diffused channel-waveguide couplers, provided, of course, that accurate data for the specific waveguide materials and design are available.

APPENDIX A: POWER TRANSFER IN A COUPLER

If a coupler is formed by two identical single-mode waveguides, the field, normalized to unit total power, can be expressed as the following superposition of the two modes of the combined system:

$$u(x, y, z) = \frac{1}{\sqrt{1 + |G|^2}} [u^+(x, z)\exp(-i\beta^+y) + Gu^-(x, z)\exp(-i\beta^-y)].$$

(A1)

Here the superscripts + and - refer to the even (symmetric) and odd (antisymmetric) modes, and $u^+(x, z)$ and $u^-(x, z)$ are the corresponding normalized mode eigenfunctions. If waveguides A and B are located, respectively, to the left and the right of the plane $x = 0$, the power in waveguide B can be expressed as

$$P_B = \int_0^\infty \int_{-\infty}^{\infty} |u(x, y, z)|^2 \, dx \, dz = \frac{\langle u^+u^- \rangle}{1 + |G|^2},$$

(A2)

where the superscript * on the angle brackets denotes integration over the right half-plane and $u^+(x, z)$ and $u^-(x, z)$ have been chosen real. (It is understood that the field vanishes for negative $z$.) It has also been assumed that the complex amplitude $G$ can be represented as $G = |G|\exp(i\phi)$.

By symmetry we must have

$$\langle u^+u^- \rangle^* = \langle u^-u^+ \rangle = 1/2.$$  

(A3)

Hence Eq. (A2) becomes

$$P_B = \frac{1}{2} + \frac{2|G|}{1 + |G|^2} \langle u^+u^- \rangle \cos(\Delta\beta y + \phi).$$

(A4)

Similarly, one obtains

$$P_A = \frac{1}{2} - \frac{2|G|}{1 + |G|^2} \langle u^+u^- \rangle \cos(\Delta\beta y + \phi),$$

(A5)

where use has been made of the relation

$$\langle u^+u^- \rangle = -\langle u^-u^+ \rangle.$$  

(A6)

We have, finally,

$$\eta(y) = \frac{P_A}{P_A + P_B} = \frac{1}{2} - \frac{2|G|}{1 + |G|^2} \langle u^+u^- \rangle \cos(\Delta\beta y + \phi).$$

(A7)

The minimum value of $\eta$ is

$$\eta_{\text{min}} = \eta(0) = \frac{1}{2} - \frac{2|G|}{1 + |G|^2} \langle u^+u^- \rangle^*,$$

(A8)

which cannot vanish unless $|G| = 1$ and $\langle u^+u^- \rangle^* = \frac{1}{2}$ simultaneously. The maximum value of $\eta$ is

$$\eta_{\text{max}} = \eta\left(\frac{\pi}{\Delta\beta}\right) \equiv \eta(\pi) = \frac{1}{2} + \frac{2|G|}{1 + |G|^2} \langle u^+u^- \rangle.$$  

(A9)

The value $\eta_{\text{max}} = 1$ will be achieved if, again, $|G| = 1$ and $\langle u^+u^- \rangle^* = \frac{1}{2}$.

To show why we cannot expect to have $\langle u^+u^- \rangle^* = \frac{1}{2}$, let us make the approximation

$$u^+(x, z) = \sqrt{2} [u_B(x, z) + u_A(x, z)],$$

$$u^-(x, z) = \sqrt{2} [u_B(x, z) - u_A(x, z)],$$

(A10)

where $u_A$ and $u_B$ are the normalized eigenfunctions for an isolated waveguide centered at the appropriate waveguide of the coupler. Then

$$\langle u^+u^- \rangle^* = \frac{1}{2} \langle u_B^2 \rangle^* - \frac{1}{2} \langle U_A^2 \rangle^*.$$  

(A11)
Fig. 13. Plot for determining coupling length when energy transfer of coupler is only 90%. Least-squares-fit straight line has a slope of 0.988.

As the waveguide separation becomes infinite, the integral in the first right-hand term approaches unity, and that in the second approaches zero. But otherwise

\[ (u^+ u^-) + < \leq \eta_{max}^2. \]  

(A12)

Equation (A7) can now be rewritten in the form

\[ \eta(y) = \eta_{min} + (2\eta_{max} - 1)\sin^2 \left( \frac{\pi y}{2l} + \phi \right), \]  

(A13)

where

\[ \eta_{min} + \eta_{max} = 1. \]  

(A14)

Figure 13 shows a plot of \( \sin^{-1} \eta_{1/2} \) versus \( \pi y/2l \) for \( \eta_{max} = 0.95 \) and \( \eta_{min} = 0.05 \), which implies that 90% of the power is transferred between waveguides. For convenience, \( \phi \) has been set equal to zero. Note that discontinuities in \( \sin^{-1} \eta_{1/2} \) occur where \( y \) is a multiple of \( l \). These occur because, according to expression (A13), \( \eta \) can never reach the values of 1 or 0, and consequently \( \sin^{-1} \eta \) can never be an integral multiple of \( \pi/2 \).

A least-squares fit of a straight line to the function \( \sin^{-1} \eta_{1/2} \), however, gives a slope of 0.988, which is very close to the slope of unity obtained when \( \eta_{max} = 1 \).

Figure 14 shows a plot of 14 points from the curve in Fig. 13 sampled at equal intervals in \( y \). The corresponding least-squares-fit straight line has the slope 0.980.

The preceding analysis justifies the use of Eqs. (1) and (2) for determining coupling lengths when a least-squares fit is applied to the experimental data.

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